# Applied Mathematics & Modeling for Chemical Engineers

## Chapter One

## Formulation of Mathematical Models

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# Definitions



- Process Modeling is the mathematical representation of a physical/chemical/biological/ etc. process and/or phenomena.

**Physical System:**

1. Heat exchanger
2. Reactor
3. Absorber
4. Distillation column
5. …

**Mathematical Equation(s):**

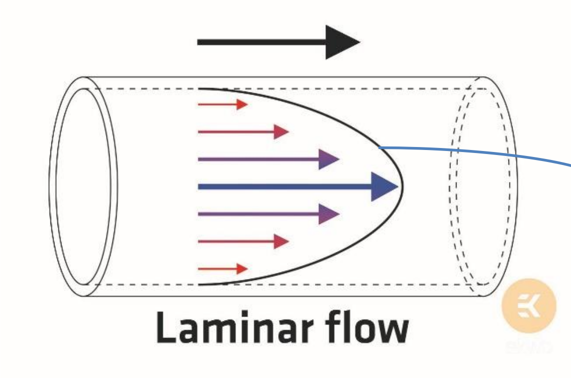
$T = f(t)$

$C = f(z,t)$

$v = f(d)$

…

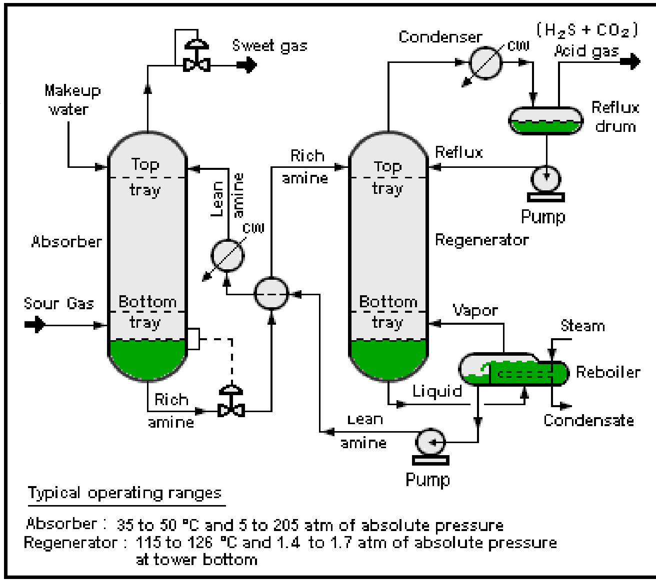
- 1960: Robert Bird introduced his textbook Transport Phenomena. → A textbook that is mainly based on problem formulation by elementary differential balances.




$$v_z = 2v_o\left[1 - \left(\frac{r}{R}\right)^2\right]$$

- **Developed Models and the need for programming language for coding**
  - C++ , MATLAB,  Simulink, etc

- **Process simulation is the use of Models or Model-based simulators to design, develop, optimize, predicate the behaviour of single equipment/integrated process.**



# Commercial Simulators

aspentech

**Aspen Plus®:** Chemical industries
**Hysys®:** Gas & Oil

Honeywell

**UniSim®:** Gas & Oil

Chemstations™

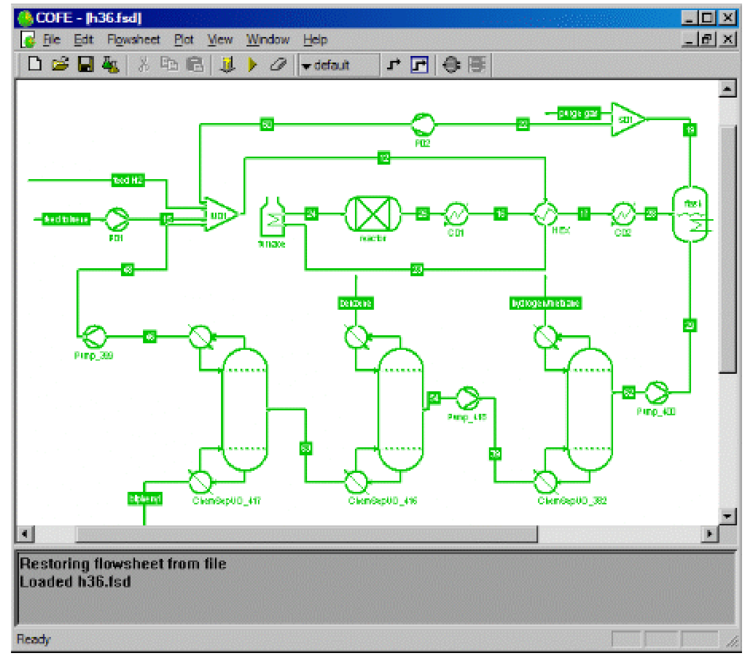
**ChemCAD®:** Chemical Industries

invensys™ Operations Management

**PRO/II®:** Chemical industries

Bryan Research & Engineering, Inc.

**ProMax®:** Gas & Oil




Intelligen, Inc.

**SuperProg Designer®:** Batch processing (Pharmaceuticals, water treatement…)

# The Four Major Stages in Process Modeling



## A. Problem Imagination

- Proper Understanding of the Problem
- Problem sketch

## B. Model Development

- Transforming our understanding into mathematical equations:
  - Purpose of the model
  - Physical & Chemical Information
  - Conservation laws and Rate laws
  - Selecting representative differential element
  - Determining the Boundary and Initial Conditions

## C. Mathematical Solution

- ❑ **Analytical**
- ❑ **Numerical**

## D. Model Validation

- ❑ **Experimental Data**
- ❑ **Operational Evidence**

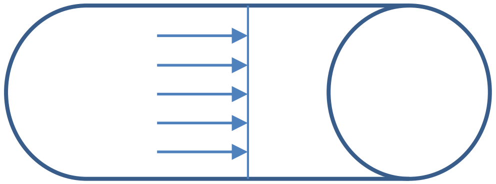
## Example: Fluid Cooling

### Case # 1

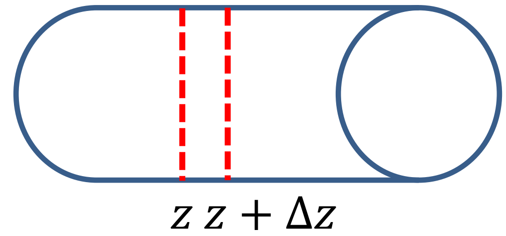
- **Develop a model to find the temperature of a fluid flowing at steady-state in a pipe, assume that the pipe wall temperature is constant and the flow is turbulent.**

- **In the solution, I will assume that ΔT is not large, this implies that ……..**

- **Turbulent flow implies that …….**



**Energy Balance on the differential element:**




$$E\Big|_{in} - E\Big|_{out} + \cancel{E\Big|_{Gen}} = \cancel{E\Big|_{acc}}$$

$$mc_pT\Big|_{z} - mc_pT\Big|_{z+\Delta z} - h(2\pi R\Delta \mathrm{z})(\mathrm{T}-\mathrm{T_w}) = 0.0$$

$$\lambda = \frac{2\pi Rh}{mc_P}$$

$$T\Big|_{z} - T\Big|_{z+\Delta z} - \lambda\Delta \mathrm{z}(\mathrm{T}-\mathrm{T_w}) = 0.0$$

**Divide by Δz and take the limit:**

$$\lim_{\Delta z\to 0}\frac{T_{z+\Delta z}-T_z}{\Delta z} = \frac{dT}{dz}$$

$$\therefore \frac{dT}{dz} = -\lambda(T-T_w)$$

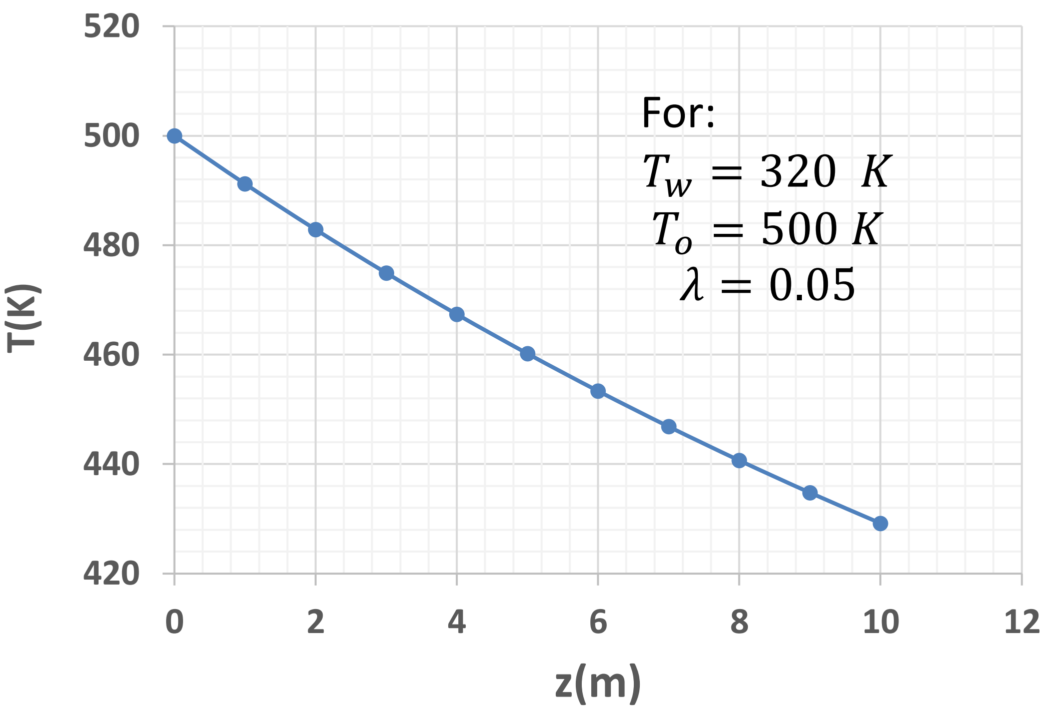
$$\frac{dT}{T - T_w} = -\lambda dz \;\rightarrow\; \int_{T_o}^{T} \frac{dT}{T - T_w} = \int_0^z -\lambda dz$$

$$\ln(T - T_w)\Big|_{T_o}^{T} = -\lambda z\Big|_0^z$$

$$\ln\left(\frac{T - T_w}{T_o - T_w}\right) = -\lambda z$$

$$\frac{T - T_w}{T_o - T_w} = \exp(-\lambda z)$$

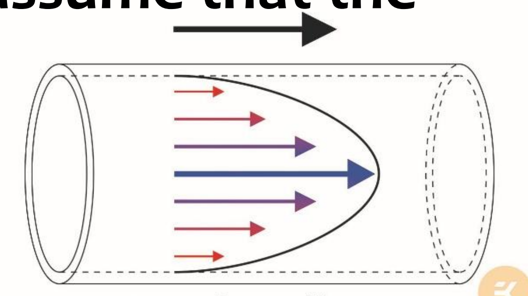
$$\boxed{T = (T_o - T_w)\exp(-\lambda z) + T_w}$$



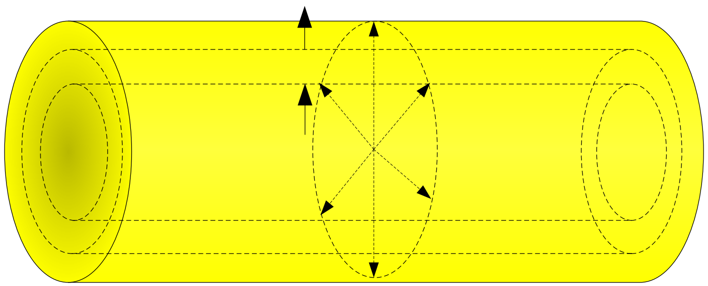
## Case # 2

- Re-model the same system, but this time assume that the flow is laminar
  - Flow pattern consequences
    - $v_z$ is not constant in the radial direction

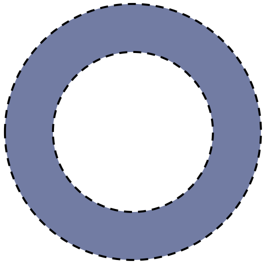
$$v_z = 2v_o\left[1 - \left(\frac{r}{R}\right)^2\right]$$




    - There will be heat conduction in the radial direction
    - Convection might be small → we have also to consider the conduction in the axial direction.
- Selection of the differential element



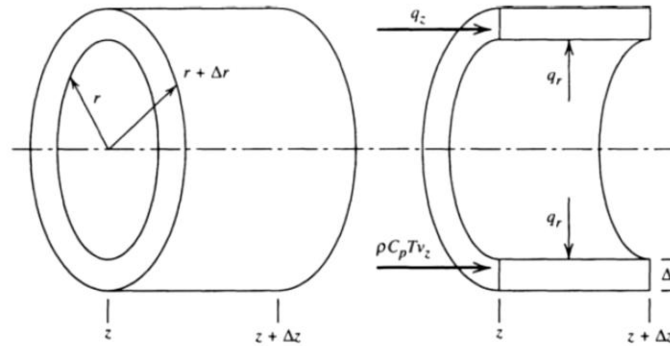
Variable changes in radial direction




Edge area

https://www.ekwb.com/

## Energy balance on the differential element




$$v_z(2\pi r\Delta r)\rho c_p T\Big|_z - v_z(2\pi r\Delta r)\rho c_p T\Big|_{z+\Delta z} + (2\pi r\Delta r)q_z\Big|_z - (2\pi r\Delta r)q_z\Big|_{z+\Delta z}$$

$$+(2\pi r\Delta \mathrm{z})\mathrm{q_r}\Big|_{\mathrm{r}} - (2\pi r\Delta \mathrm{z})\mathrm{q_r}\Big|_{\mathrm{r}+\Delta r} = 0.0$$

## Divide by $2\pi\Delta r\Delta z$ & take the limit

$$-v_z r\rho c_p \frac{T|_{z+\Delta z} - T|_z}{\Delta \mathrm{z}} - \frac{rq_z|_{z+\Delta z} - rq_z|_z}{\Delta z} - \frac{rq_r|_{r+\Delta r} - rq_r|_r}{\Delta r} = 0.0$$

$$-v_z r\rho c_p \frac{\partial T}{\partial \mathrm{z}} - \frac{\partial(rq_z)}{\partial z} - \frac{\partial(rq_r)}{\partial r} = 0.0$$

## Divide by r & rearrange

$$v_z\rho c_p \frac{\partial T}{\partial \mathrm{z}} + \frac{\partial q_z}{\partial z} + \frac{1}{r}\frac{\partial(rq_r)}{\partial r} = 0.0$$

**We know that**

$$q_r = -k\frac{\partial T}{\partial r} \quad \textbf{And} \quad q_z = -k\frac{\partial T}{\partial z}$$

$$v_z \rho c_p \frac{\partial T}{\partial z} - \frac{\partial\left(k\frac{\partial T}{\partial z}\right)}{\partial z} - \frac{1}{r}\frac{\partial\left(rk\frac{\partial T}{\partial r}\right)}{\partial r} = 0.0$$

$$v_z \rho c_p \frac{\partial T}{\partial z} = k\frac{\partial^2 \mathrm{T}}{\partial z^2} + \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$2v_o\left[1-\left(\frac{r}{R}\right)^2\right]$$

**Boundary Conditions**

$$z = 0 \;\&\; r = r \rightarrow T(0, r) = T_o$$

$$z = \infty \;\&\; r = r \rightarrow T(\infty, r) = T_w$$

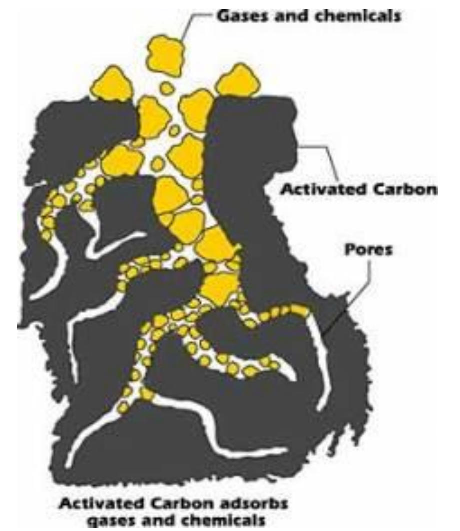
$$z = z \;\&\; r = R \rightarrow T(z, R) = T_w$$

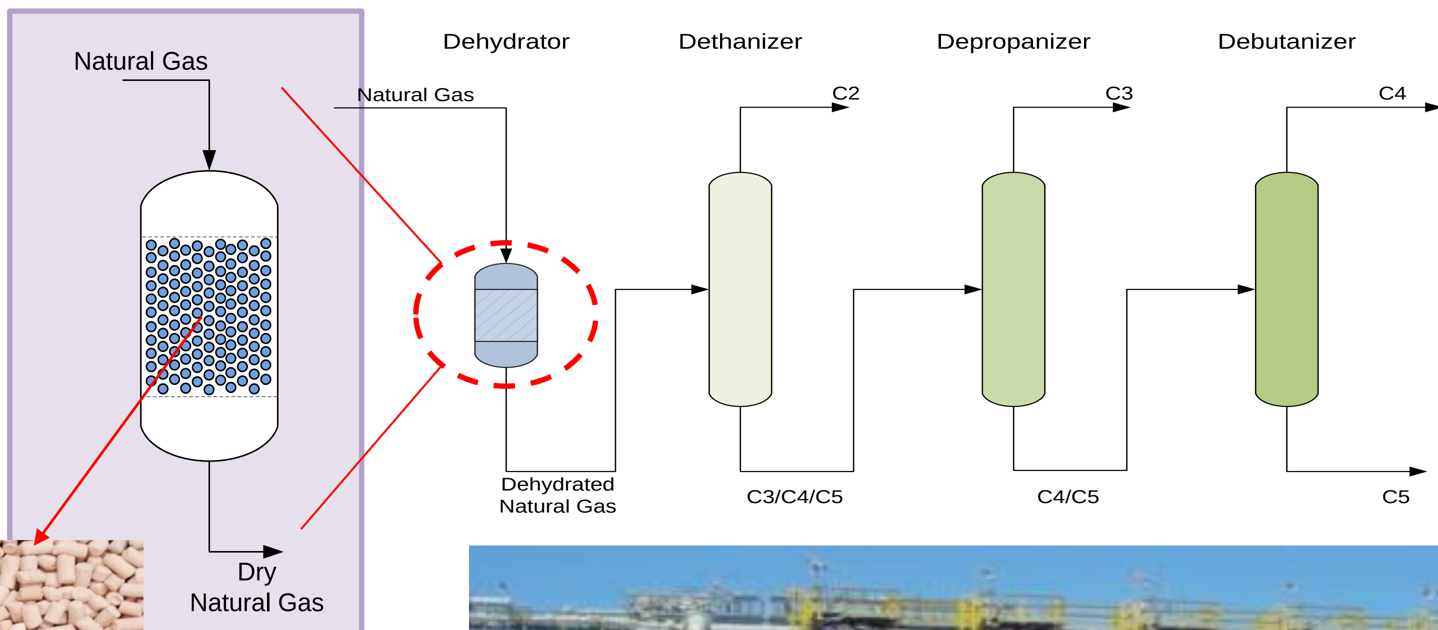
$$z = z \;\&\; r = r \rightarrow \frac{\partial T}{\partial r} = 0$$

$$T(z, r) = \cdots$$

## Example: Adsorption Bed

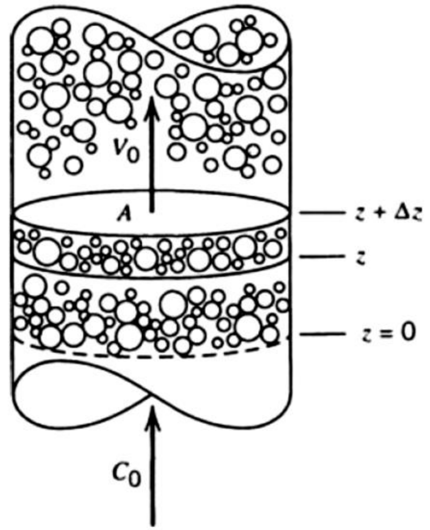
- L-S or G-S separation
- Could be chemical or physical, what is the difference?
- Activated Carbon is a good adsorbent, why?





- **Objective: Develop a model to find the concentration of the adsorbate within the adsorption bed as function of time and location?**




- **The model to be developed based on:**
  - **A differential element $A_c \Delta z$**
  - **Isothermal operation**
  - **Flat velocity profile ($v_o = constant$)**
  - **No axial diffusion**

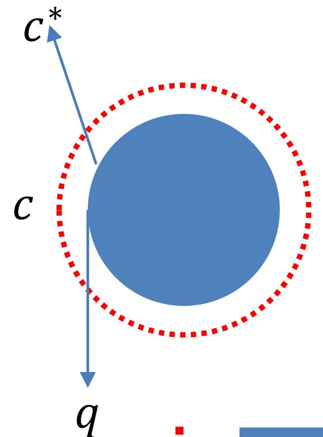
**Balance on the adsorbate**

$$v_o A_c c \Big|_z - v_o A_c c \Big|_{z+\Delta z} = \varepsilon A_c \Delta z \frac{\partial c}{\partial t} + (1-\varepsilon) A_c \Delta z \frac{\partial q}{\partial t}$$

**Divide by $A_c \Delta z$ and take the limit**

$$-v_o \frac{\partial c}{\partial z} = \varepsilon \frac{\partial c}{\partial t} + (1-\varepsilon) \frac{\partial q}{\partial t}$$

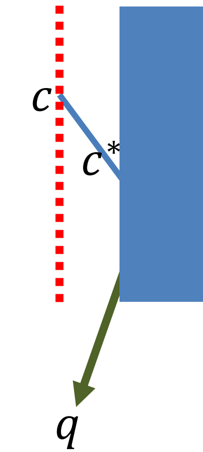
**Based on the interfacial equilibrium**




$$q = Kc^*$$

$$\boxed{-v_o \frac{\partial c}{\partial z} = \varepsilon \frac{\partial c}{\partial t} + (1-\varepsilon) K \frac{\partial c^*}{\partial t}} \quad \dots\dots..(1)$$

**Balance on the solid phase:**




$$k_c a A_c \Delta z (c - c^*) = A_c \Delta z (1-\varepsilon) \frac{\partial q}{\partial t}$$

**Divide by $A_c \Delta z$ and use** $q = Kc^*$

$$\boxed{k_c a (c - c^*) = (1-\varepsilon) K \frac{\partial c^*}{\partial t}} \quad \dots\dots..(2)$$

**Boundary and Initial Conditions**

$$z = z \;\&\; t = 0 \quad \rightarrow c^*(z, 0) = 0$$

$$z = z \;\&\; t = 0 \quad \rightarrow c(z, 0) = 0$$

$$z = 0 \;\&\; t = t \quad \rightarrow c(0, t) = \mathrm{c_o}$$

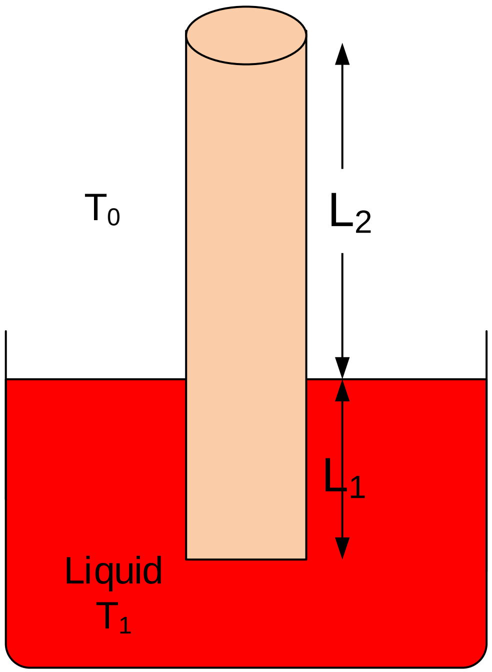
# Model Details



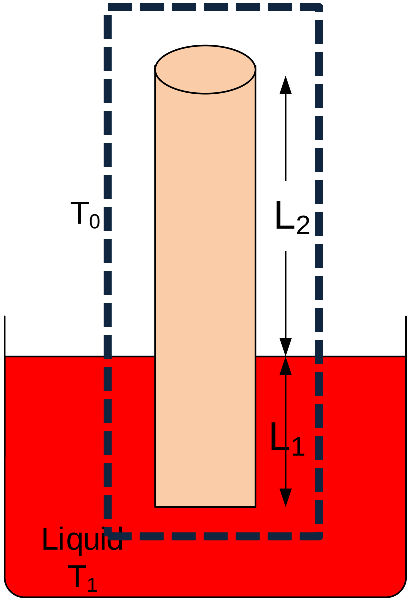
- **Objective: Estimating the heat removal from a solvent bath by a rod.**
- **General Assumptions:**
  - Ignore heat transfer at rod ends
  - Overall heat transfer coefficient is constant
  - No solvent evaporation
  - The system is at st.st.

**Level # I**

**Assume the rod temperature is uniform inside and outside the solvent (i.e. no temperature gradient in the radial or axial directions)**



## Energy balance around the rod




$$E\Big|_{in} = E\Big|_{out}$$

$$h_L(2\pi R L_1)(T_1 - T) = h_G(2\pi R L_2)(T - T_o)$$

**Divide by** $2\pi R$

$$h_L L_1 T_1 - h_L L_1 T = h_G L_2 T - h_G L_2 T_o$$

$$h_L L_1 T_1 + h_G L_2 T_o = h_G L_2 T + h_L L_1 T$$

**Solve for T**

$$\boxed{T = \frac{h_L L_1 T_1 + h_G L_2 T_o}{h_G L_2 + h_L L_1}}$$

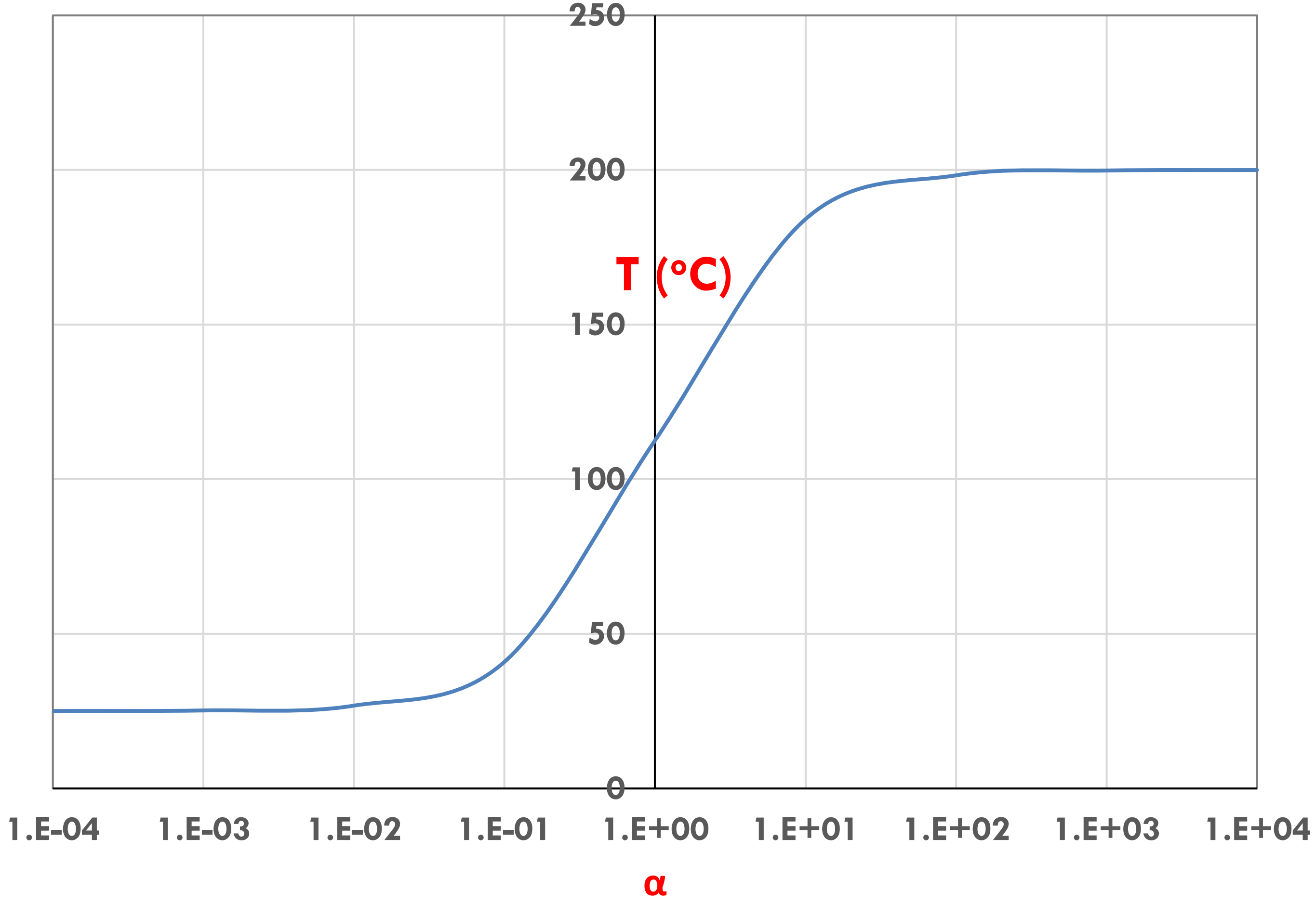
**Define** $\alpha$

$$\alpha = \frac{h_L L_1}{h_G L_2}$$

$$\boxed{\rightarrow T = \frac{\alpha T_1 + T_o}{1 + \alpha}}$$

**Temperature of the rod as function of α for a system at**

$$T_1 = 200\ {}^oC\ and\ T_o = 25{}^oC$$



**if $\boldsymbol{\alpha} \gg \mathbf{1}$**

$$T = \frac{\alpha T_1 + T_o}{1+\alpha} = \frac{\alpha T_1}{1+\alpha} + \frac{T_o}{1+\alpha}$$

$$\rightarrow T = T_1 + \frac{T_o}{1+\alpha}$$

**Relatively small**

**This means that the temperature of the rod is very close to the temperature of the liquid**

## Rate of heat transfer

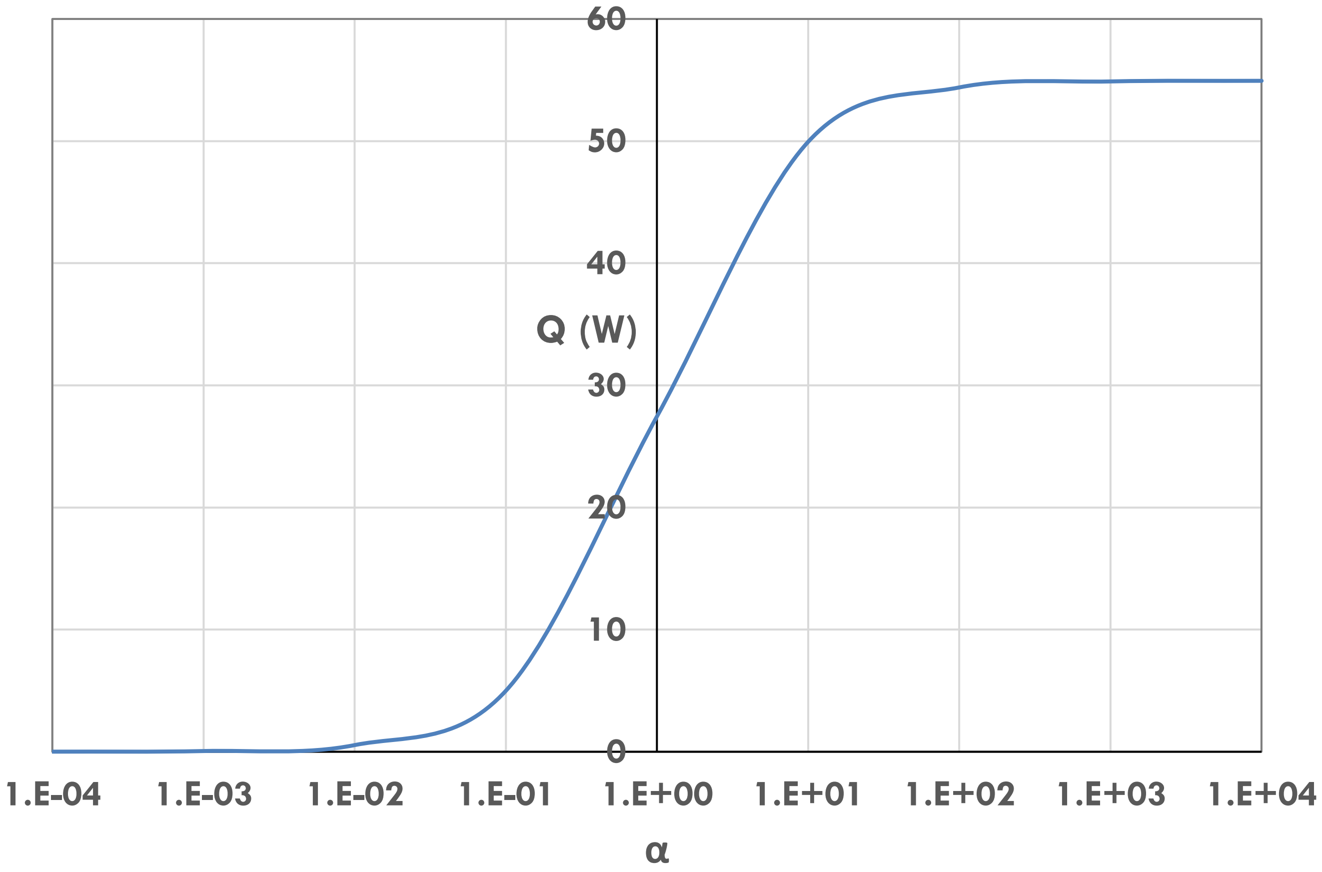
$$Q = h_G(2\pi R L_2)(\mathrm{T} - \mathrm{T_o})$$

**if $\boldsymbol{\alpha} \gg \mathbf{1} \ \rightarrow \boldsymbol{T} \cong \boldsymbol{T_1}$**

$$Q \cong h_G(2\pi R L_2)(\mathrm{T_1} - \mathrm{T_o})$$

## Heat loss from the system as function of α for a system at

$$T_1 = 200\ ^oC, T_o = 25^oC, h_G = 10\,\frac{W}{m^2.K}, R = 0.05\ m\ and\ L_2 = 0.1\ m$$

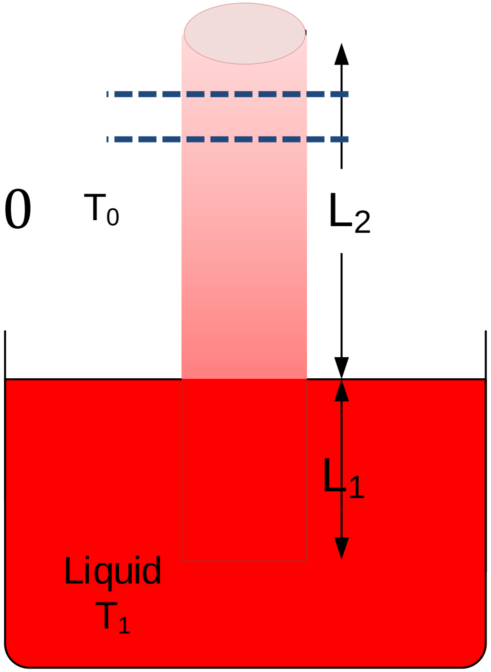


## Level # 2

- **The same as Level # 1, but at this level we will assume that:**
  - The temperature of the rod in the liquid is uniform and equals the temperature of the liquid.
  - The temperature of the rod outside the liquid is not uniform in the axial direction.

**Energy balance on the differential element**

$$\pi R^2 q\Big|_x - \pi R^2 q\Big|_{x+\Delta x} - h_G(2\pi R\Delta x)(T - T_o) = 0$$




**Divide by** $\pi R^2 \Delta x$ **And take the limit**

$$-\frac{dq}{dx} - \frac{2h_G}{R}(T - T_o) = 0$$

**But** $q = -k\dfrac{dT}{dx}$

$$k\frac{d^2T}{dx^2} = \frac{2h_G}{R}(T - T_o)$$

- **ODE**
- **2nd Order**
- **Non-homogeneous**

**Boundary Conditions:**

$$x = 0 \;\rightarrow T = T_1$$

$$x = L_2 \;\rightarrow \left.\frac{dT}{dx}\right|_{L_2} = 0$$

$$\frac{d^2T}{dx^2} - \lambda(T - T_o) = 0$$

$$\lambda = \frac{2h_G}{kR}$$

$$\theta = T - T_o$$

$$d\theta = dT \;\; \& \; d^2\theta = d^2T$$

$$\frac{d^2\theta}{dx^2} - \lambda\theta = 0$$

$$m^2 - \lambda = 0 \quad \rightarrow m = \pm\sqrt{\lambda}$$

$$\theta(x) = A\cosh\left(\sqrt{\lambda}x\right) + B\sinh(\sqrt{\lambda}x)$$

**1st BC**

$$x = 0 \ \rightarrow T = T_1 \rightarrow \theta = T_1 - T_o$$

$$T_1 - T_o = A\cosh(0) + B\sinh(0)$$

$$\rightarrow A = T_1 - T_o$$

**2nd BC**

$$x = L_2 \ \rightarrow \frac{dT}{dx}\bigg|_{L_2} = 0 \rightarrow \frac{d\theta}{dx}\bigg|_{L_2} = 0$$

$$\frac{d\theta}{dx} = A\sqrt{\lambda}\sinh\left(\sqrt{\lambda}x\right) + B\sqrt{\lambda}\cosh(\sqrt{\lambda}x)$$

$$\rightarrow \ 0 = (T_1 - T_o)\sqrt{\lambda}\sinh\left(\sqrt{\lambda}L_2\right) + B\sqrt{\lambda}\cosh(\sqrt{\lambda}L_2)$$

$$\rightarrow B = -\frac{(T_1 - T_o)\sinh\left(\sqrt{\lambda}L_2\right)}{\cosh\left(\sqrt{\lambda}L_2\right)}$$

$$\theta(x) = (T_1 - T_o)cosh\left(\sqrt{\lambda}x\right) - \frac{(T_1 - T_o)sinh\left(\sqrt{\lambda}L_2\right)}{cosh\left(\sqrt{\lambda}L_2\right)} sinh(\sqrt{\lambda}x)$$

$$T(x) - T_o = (T_1 - T_o)cosh\left(\sqrt{\lambda}x\right) - (T_1 - T_o)tanh\left(\sqrt{\lambda}L_2\right)sinh(\sqrt{\lambda}x)$$

$$\boxed{T(x) = T_o + (T_1 - T_o)cosh\left(\sqrt{\lambda}x\right) - (T_1 - T_o)tanh\left(\sqrt{\lambda}L_2\right)sinh(\sqrt{\lambda}x)}$$

**For** $k \gg h_G \rightarrow \lambda \ll 1 \quad and\ x = L_2$

$$T(L_2) = T_o + (T_1 - T_o)cosh(\sim 0) - (T_1 - T_o)tanh(\sim 0)sinh(\sim 0)$$

$$T(L_2) = T_o + (T_1 - T_o) - (T_1 - T_o)(0)$$

$$T(L_2) = T_1$$

**Compare to level 1**

**For** $k \ll h_G \rightarrow \lambda \gg 1 \quad and\ x = L_2$

$$T(L_2) = T_o + (T_1 - T_o)cosh\left(\sqrt{\lambda}L_2\right) - (T_1 - T_o)tanh\left(\sqrt{\lambda}L_2\right)sinh(\sqrt{\lambda}L_2)$$

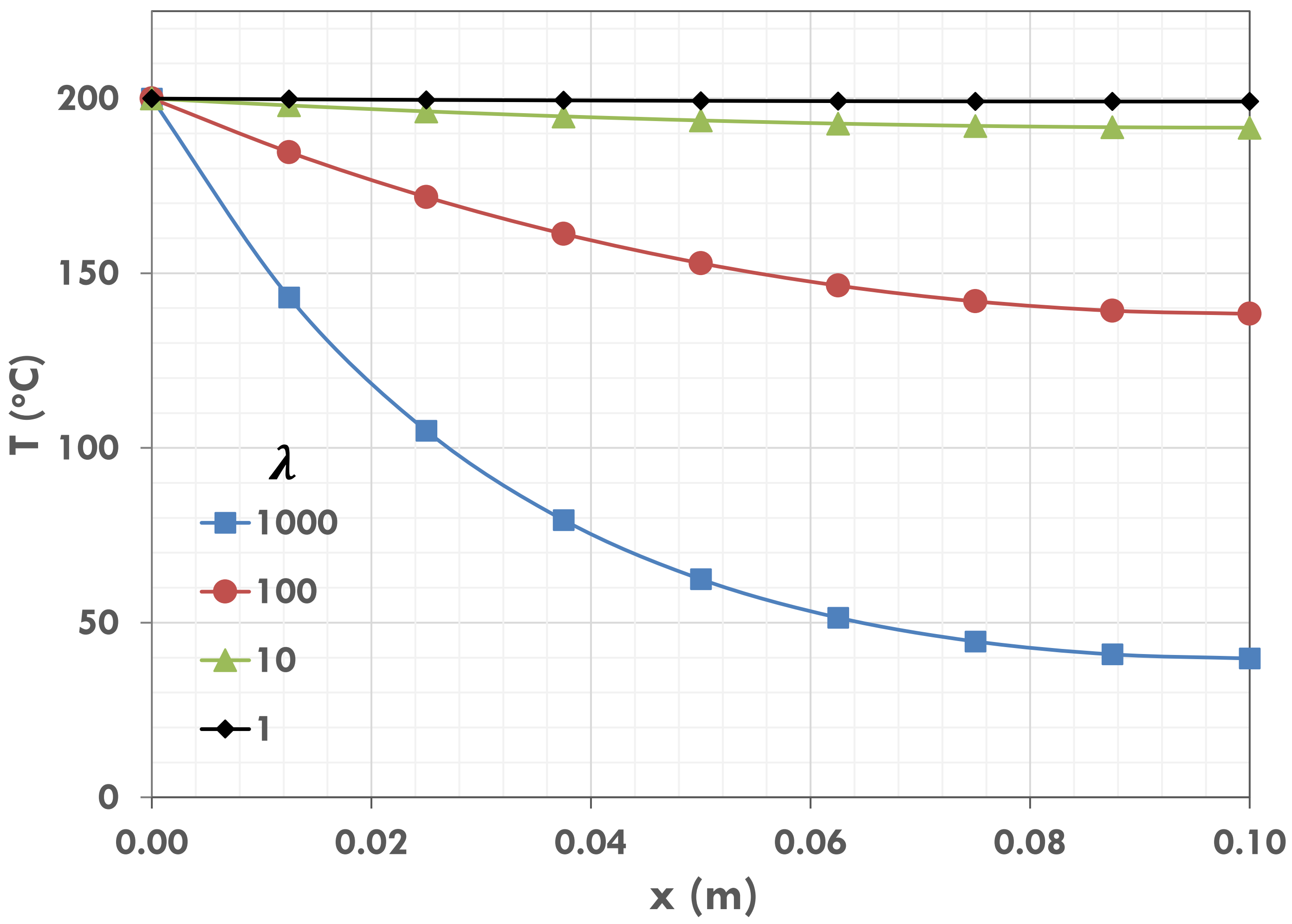
$$T(L_2) = T_o + (T_1 - T_o)cosh\left(\sqrt{\lambda}L_2\right) - \frac{(T_1 - T_o)sinh\left(\sqrt{\lambda}L_2\right)}{cosh\left(\sqrt{\lambda}L_2\right)}sinh(\sqrt{\lambda}L_2)$$

$$T(L_2) = T_o + (T_1 - T_o)\left(\frac{cosh\left(\sqrt{\lambda}L_2\right)cosh\left(\sqrt{\lambda}L_2\right) - sinh\left(\sqrt{\lambda}L_2\right)\sinh(\sqrt{\lambda}L_2)}{cosh\left(\sqrt{\lambda}L_2\right)}\right)$$

$$T(L_2) = T_o + (T_1 - T_o)\left(\frac{1}{cosh\left(\sqrt{\lambda}L_2\right)}\right)$$

**Very small**

$$\rightarrow T(L_2) = T_o$$



T (°C)
200
150
100
50
0
λ
1000
100
10
1
0.00
0.02
0.04
0.06
0.08
0.10
x (m)

## Rate of heat transfer

Option #1 $$Q = -k\pi R^2 \frac{dT}{dx}\bigg|_{x=0}$$

$$\frac{dT}{dx} = (T_1 - T_o)\sqrt{\lambda}sinh\left(\sqrt{\lambda}x\right) - (T_1 - T_o)\sqrt{\lambda}tanh\left(\sqrt{\lambda}L_2\right)cosh\left(\sqrt{\lambda}x\right)$$

$$\frac{dT}{dx}\bigg|_{x=0} = (T_1 - T_o)\sqrt{\lambda}sinh(0) - (T_1 - T_o)\sqrt{\lambda}tanh\left(\sqrt{\lambda}L_2\right)cosh(0)$$

$$\frac{dT}{dx}\bigg|_{x=0} = -(T_1 - T_o)\sqrt{\lambda}tanh\left(\sqrt{\lambda}L_2\right)$$

$$\rightarrow Q = k\pi R^2(T_1 - T_o)\sqrt{\lambda}tanh\left(\sqrt{\lambda}L_2\right) \qquad \times\frac{\sqrt{\lambda}L_2}{\sqrt{\lambda}\mathrm{L}_2}$$

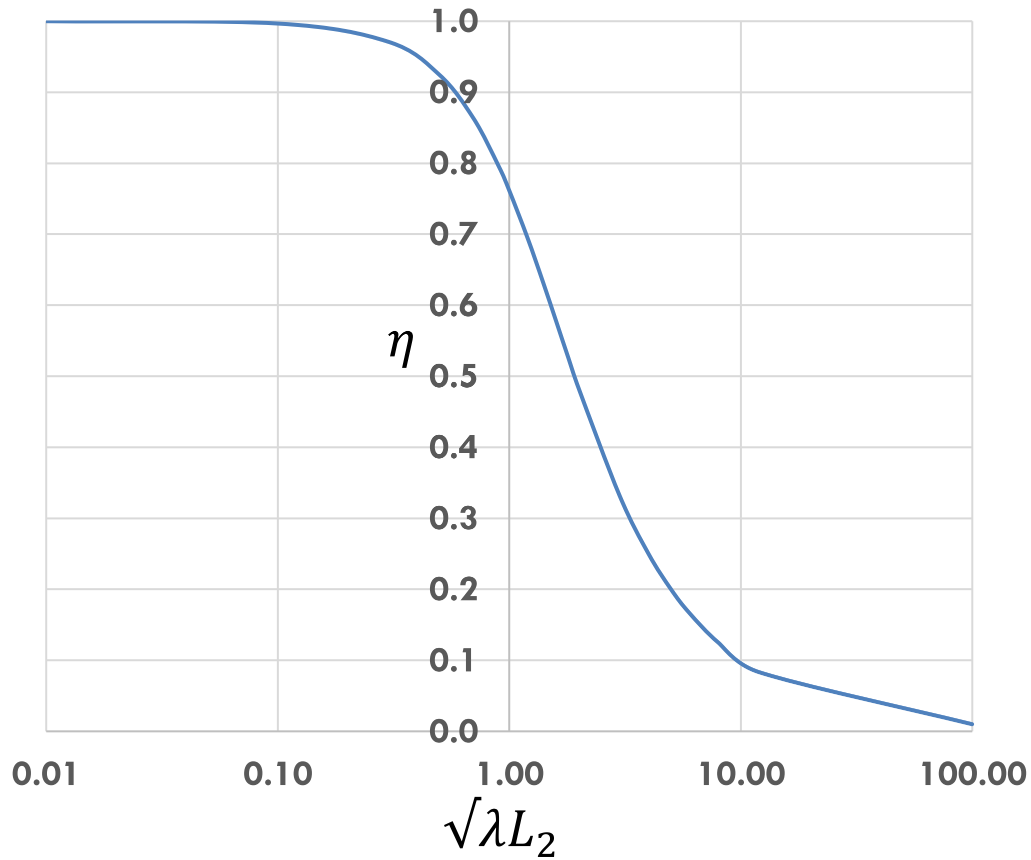
$$Q = k\pi R^2(T_1 - T_o)\lambda L_2 \frac{tanh\left(\sqrt{\lambda}L_2\right)}{\sqrt{\lambda}L_2}$$

$$Q = k\pi R^2(T_1 - T_o)\left(\frac{2h_G}{Rk}\right) L_2 \frac{tanh\left(\sqrt{\lambda}L_2\right)}{\sqrt{\lambda}L_2}$$

$$\therefore Q = (2\pi R L_2) h_G (T_1 - T_o) \frac{\tanh\left(\sqrt{\lambda} L_2\right)}{\sqrt{\lambda} L_2}$$

**Effectiveness factor**

$$\eta = \frac{\tanh\left(\sqrt{\lambda} L_2\right)}{\sqrt{\lambda} L_2}$$




Option #2

$$Q = \int_0^{L_2} h_G (2\pi R)(T - T_o)\, dx$$

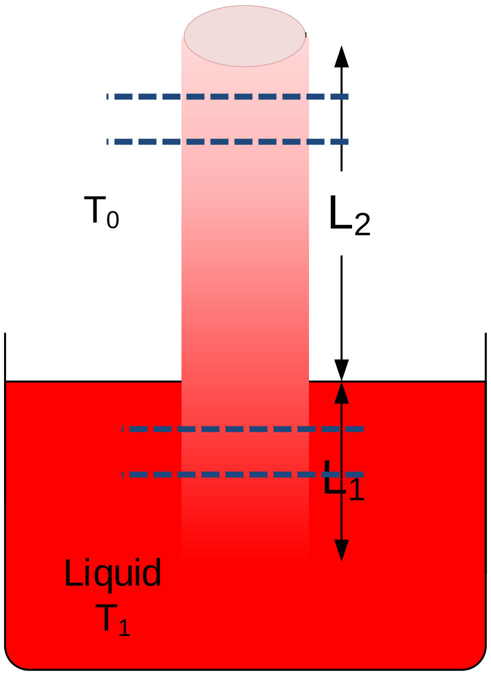
Function of x

# Level # 3

**At this level we will keep the same previous assumptions except that there are temperature gradients in the rod within the liquid and within the gas phase.**

$T_L$: Temperature of the rod in the lower part

$T_U$: Temperature of the rod in the upper part




**We have to develop 2 models, one for the upper section of the rod and one for the lower section**

$$k\frac{d^2T_L}{dx^2} = \frac{2h_L}{R}(T_L - T_1)$$

$$k\frac{d^2T_U}{dx^2} = \frac{2h_U}{R}(T_U - T_o)$$

## Boundary Conditions

$$x = -L_1 \quad \rightarrow \frac{dT_L}{dx} = 0$$

$$x = L_2 \quad \rightarrow \frac{dT_U}{dx} = 0$$

$$x = 0 \quad \rightarrow T_L = T_U$$

$$x = 0 \rightarrow \left.\frac{dT_L}{dx}\right|_{x=0} = \left.\frac{dT_U}{dx}\right|_{x=0}$$

## Solution

$$T_L = T_1 + A\cosh\big(n(x + L_1)\big)$$

$$T_U = T_o + B\cosh\big(m(L_2 - x)\big)$$

$$n = \sqrt{\frac{2h_L}{Rk}} \qquad\qquad m = \sqrt{\frac{2h_G}{Rk}}$$

$$A = \frac{-(T_1 - T_o)}{\cosh(nL_1) + \frac{n}{m}\frac{\sinh(nL_1)}{\sinh(mL_2)}\cosh(mL_2)}$$

$$B = \frac{(T_1 - T_o)}{\cosh(mL_2) + \frac{m}{n}\frac{\sinh(nL_1)}{\sinh(mL_2)}\cosh(mL_2)}$$

## Rate of heat transfer

$$Q = -\pi R^2 k \frac{dT_L}{dx}\Big|_{x=0}$$

$$T_L = T_1 + Acosh\big(n(x + L_1)\big)$$

$$\frac{dT_L}{dx} = nAsinh\big(n(x + L_1)\big)$$

$$\frac{dT_L}{dx}\Big|_{x=0} = nAsinh\big(n(0 + L_1)\big) = nAsinh(nL_1)$$

$$Q = -\pi R^2 knA sinh(nL_1)$$

$$Q = -\pi R^2 kn sinh(nL_1) \frac{-(T_1 - T_o)}{\cosh(nL_1) + \frac{n}{m}\frac{\sinh(nL_1)}{\sinh(mL_2)}\cosh(mL_2)}$$

**Divide by** $sinh(nL_1)$

$$Q = \pi R^2 kn \frac{(T_1 - T_o)}{\frac{\cosh(nL_1)}{\sinh(nL_1)} + \frac{n}{m}\frac{\cosh(mL_2)}{\sinh(mL_2)}}$$

$$Q = \pi R^2 kn \frac{(T_1 - T_o)}{\frac{1}{\tanh(nL_1)} + \frac{n}{m}\frac{1}{\tanh(mL_2)}}$$

**Multiply by:** $\frac{m}{n}\tanh(mL_2) \,/\, \frac{m}{n}\tanh(mL_2)$

$$Q = \pi R^2 k m \frac{(T_1 - T_o)\tanh(mL_2)}{\dfrac{m}{n}\dfrac{\tanh(mL_2)}{\tanh(nL_1)} + 1}$$

**Multiply by:** $\dfrac{mL_2}{mL_2}$

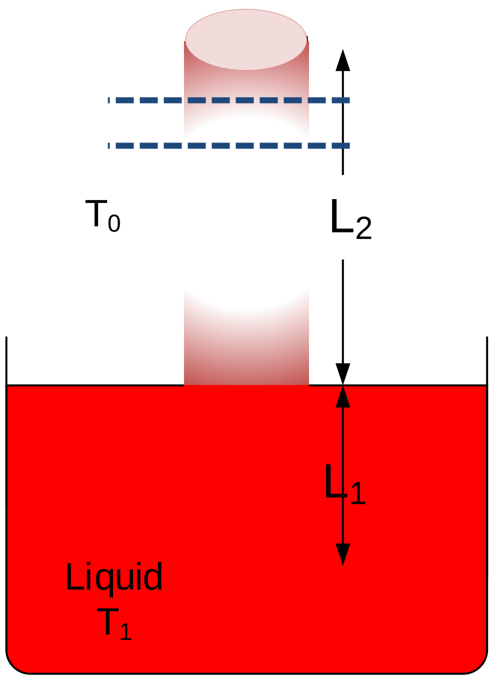
$$Q = \pi R^2 k \boxed{m^2} L_2 \frac{(T_1 - T_o)}{\dfrac{m}{n}\dfrac{\tanh(mL_2)}{\tanh(nL_1)} + 1} \underbrace{\frac{\tanh(mL_2)}{mL_2}}_{\eta}$$

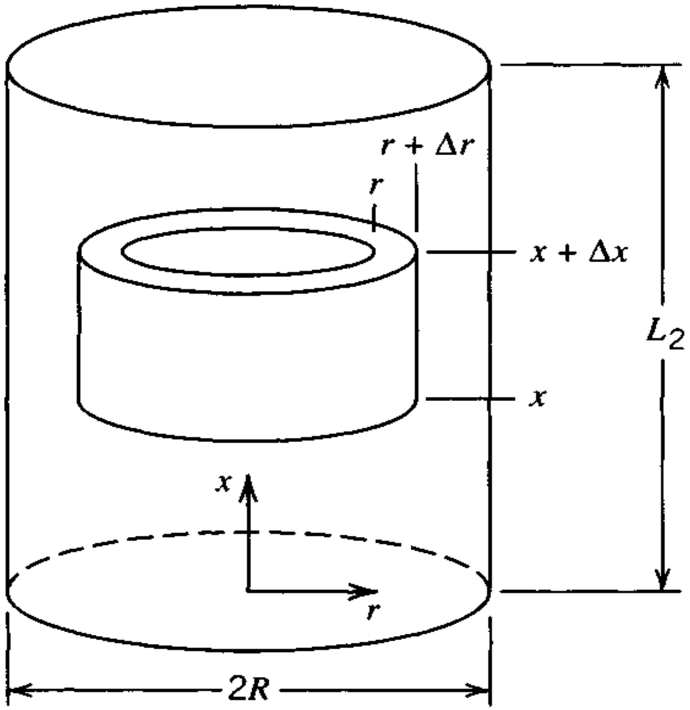
$$m^2 = \lambda = \frac{2h_G}{Rk}$$

$$Q = h_G(2\pi R L_2)\eta \frac{T_1 - T_o}{1 + \left(\dfrac{m}{n}\dfrac{tanh(mL_2)}{tanh(nL_1)}\right)}$$

# Level # 4

**At this level we will consider the heat flux in the radial direction for the upper section.**




$$(2\pi r\Delta r)q_x\Big|_x - (2\pi r\Delta r)q_x\Big|_{x+\Delta x} + (2\pi r\Delta x)q_r\Big|_r - (2\pi r\Delta x)q_r\Big|_{r+\Delta r} = 0$$

**Divide by** $2\pi\Delta r\Delta x$ **and take the limit**

$$-\frac{\partial(rq_x)}{\partial x}-\frac{\partial(rq_r)}{\partial r}=0.0$$

$$q_r=-k\frac{\partial T}{\partial r} \qquad q_x=-k\frac{\partial T}{\partial x}$$

$$kr\frac{\partial^2 T}{\partial x^2}+k\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=0.0$$

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right)=0.0$$

**Boundary Conditions:**

$$r=0\ and\ x=x\ \rightarrow \left.\frac{\partial T}{\partial r}\right|_{r=0}=0$$

$$r=R\ and\ x=x\ \rightarrow -k\frac{\partial T}{\partial r}=h_G(T-T_o)$$

$$r = r \; and \; x = 0 \;\; \rightarrow T = T_1$$

$$r = r \; and \; x = L_2 \;\; \rightarrow \frac{\partial T}{\partial x} = 0$$

$$Q = \frac{2\pi R^2 k (T_1 - T_o)}{L_2 \Delta} \sum_{n=1}^{\infty} \frac{\beta_n < 1, K_n >^2}{< K_n, K_n >} \tanh\left(\frac{\beta_n}{\Delta}\right)$$

$$Bi = \frac{h_G R}{k}$$

When$Bi \ll 1$

$$\boxed{\therefore Q = (2\pi R L_2) h_G (T_1 - T_o) \eta}$$

# Boundary Conditions



- **Homogenous BCs**

$$y(x) = 0 \quad @\, x = x_o$$

$$\left.\frac{dy}{dx}\right|_{x=x_o} = 0 \quad @\, x = x_o$$

$$\beta y + \frac{dy}{dx} = 0 \quad @\, x = x_o$$

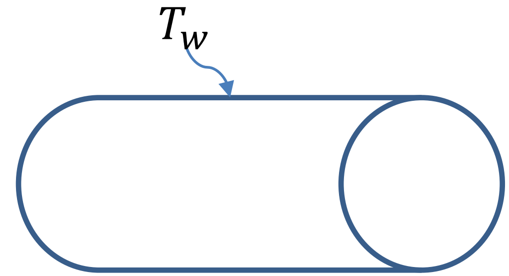
- **Nonhomogeneous BCs**

$$y(x) = \alpha \quad @\, x = x_o$$

- **Nonhomogeneous BCs can be transferred to homogenous BCs**

# Example

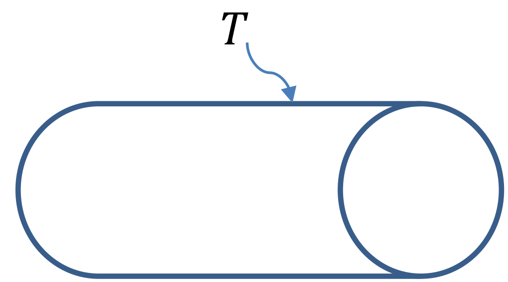
**Convert the following nonhomogeneous BCs into homogenous BCs**




$$a)\ T(z,R) = T_W$$

Assume $\theta = T - T_w$

$$\rightarrow \theta(z,R) = T_w - T_w = 0$$




$$b)\ if\ at\ r = R \quad -k\frac{dT}{dr} = U(T - T_c)$$

Assume $\theta = T - T_c$

$$-k\frac{d\theta}{dr} = U\theta \ \Rightarrow \frac{U}{k}\theta + \frac{d\theta}{dr} = 0$$